6) Find $f'(x)$ where $f(x) = \sin(٥x^{٢}-٣)$

g(x) will be sin(h(x)) and h(x) will be $(5x^{2}-٣)$

g'(h(x)) = $\cos(5x^{2}-3)$

h'(x) = 10x

f'(x) = g'(h(x)) • h'(x)

f'(x) = $10x\cos(5x^{2}-3)$ <-- solution

7) find $f'(x)$ where $f(x) = \sqrt{\cos 4x-2x}$

g(x) = $\sqrt{x}$
h(x) = cos 4 x – 2x

g'(h(x)) = $\dfrac{1}{2\sqrt{\cos 4x-2x}}$

h'(x) = - 4 sin 4x – 2

f'(x) = g'(h(x)) • h'(x) = $\dfrac{1}{2\sqrt{\cos 4x-2x}}\times(-٤\sin 4x-2) = \dfrac{-4\sin 4x-2}{2\sqrt{\cos ٤x-2x}}$ **<-- solution**

8)

f(x) = ln 2x

f'(x) = g'(h(x))• h'(x)

g(x) = ln x
h(x) = 2x

g'(x) = 1/x
h'(x) = 2

g'(h(x)) = 1/(2x)

f'(x) = g'(h(x))• h'(x) = 1/(2x) • 2 = 2/(2x) = **1/x** **<--- solution**

13)

$$y=(\text{٥}x^2-2x)^{e^x}$$

ln both sides:

$$\ln y=e^x\ln(5x^2-2x)$$

breaking this down for product and chaining rules:

g(x) = e^x
h(x) = k(u(x))

k(x) = ln (x)
u(x) = $(5x^{\text{٢}}-\text{٢}x)$

so we differentiate both sides (product rule first):

$$\frac{\text{١}}{y}\frac{dy}{dx}=g'(x)h(x)+h'(x)g(x)$$

now the chain rule on h'(x):

$$\frac{1}{y}\frac{dy}{dx}=g'(x)h(x)+k'(u(x))u'(x)g(x)$$

multiply both sides by y:

$$\frac{dy}{dx}=(g'(x)h(x)+k'(u(x))u'(x)g(x))\times y$$

Now we differentiate:

g(x) = e^x
g'(x) = e^x -- easy enough

k(x) = ln (x)
k'(x) = 1/x -- also easy

u(x) = $(5x^2-2x)$
u'(x) = $(10x-2)$ -- simple (the power rule is my friend)

now we plug it all in:

from above so I don't have to keep scrolling up:

$$y=(٥x^{٢}-٢x)^{e^x}$$

h(x) = k(u(x)) = $\ln(5x^2-٢x)$

$$\frac{dy}{dx}=(g'(x)h(x)+k'(u(x))u'(x)g(x))\times y$$

$$\frac{dy}{dx}=(\overset{g'(x)}{e^x}\times\overset{h(x)}{\ln(5x^2-2x)}+\overset{k'(u(x))}{\frac{1}{5x^2-2\mathrm{x}}}\times\overset{u'(x)}{(10\mathrm{x}-2)}\times\overset{g(x)}{e^x})\times\overset{y}{(5x^2-2x)^{e^x}}$$

Which is the solution. To move it to look like the answer on the test, we rearrange to put y in front, flip the addition, and combine 1/u(x) and (u'(x) times g(x)):

$$\frac{dy}{dx}=(5x^2-2x)^{e^x}\times(\frac{e^x(10\mathrm{x}-2)}{5x^2-2\mathrm{x}}+e^x\times\ln(5x^2-2x))$$

finally (phew) we pull out e^x as a common factor and put it in front, getting the same solution as A on the quiz:

$$\frac{dy}{dx}=e^x(5x^2-2x)^{e^x}(\frac{10\mathrm{x}-2}{5x^2-2\mathrm{x}}+\ln(5x^2-2x))$$