

6) Find $f'(x)$ where $f(x) = \sin(5x^2 - 3)$

$g(x)$ will be $\sin(h(x))$ and $h(x)$ will be $(5x^2 - 3)$

$$g'(h(x)) = \cos(5x^2 - 3)$$

$$h'(x) = 10x$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$f'(x) = 10x \cos(5x^2 - 3) \quad \leftarrow \text{solution}$$

7) find $f'(x)$ where $f(x) = \sqrt{\cos 4x - 2}$

$$g(x) = \sqrt{x}$$

$$h(x) = \cos 4x - 2$$

$$g'(h(x)) = \frac{1}{2\sqrt{\cos 4x - 2}}$$

$$h'(x) = -4 \sin 4x$$

$$f'(x) = g'(h(x)) \cdot h'(x) = \frac{1}{2\sqrt{\cos 4x - 2}} \times (-4 \sin 4x) = \frac{-4 \sin 4x}{2\sqrt{\cos 4x - 2}} \quad \leftarrow \text{solution}$$

8)

$$f(x) = \ln 2x$$

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$g(x) = \ln x$$

$$h(x) = 2x$$

$$g'(x) = 1/x$$

$$h'(x) = 2$$

$$g'(h(x)) = 1/(2x)$$

$$f'(x) = g'(h(x)) \cdot h'(x) = 1/(2x) \cdot 2 = 2/(2x) = 1/x \quad \leftarrow \text{solution}$$

13)

$$y = (5x^2 - 2x)e^x$$

ln both sides:

$$\ln y = e^x \ln(5x^2 - 2x)$$

breaking this down for product and chaining rules:

$$g(x) = e^x$$

$$h(x) = k(u(x))$$

$$k(x) = \ln(x)$$

$$u(x) = (5x^2 - 2x)$$

so we differentiate both sides (product rule first):

$$\frac{1}{y} \frac{dy}{dx} = g'(x)h(x) + h'(x)g(x)$$

now the chain rule on h'(x):

$$\frac{1}{y} \frac{dy}{dx} = g'(x)h(x) + k'(u(x))u'(x)g(x)$$

multiply both sides by y:

$$\frac{dy}{dx} = (g'(x)h(x) + k'(u(x))u'(x)g(x)) \times y$$

Now we differentiate:

$$g(x) = e^x$$

$$g'(x) = e^x \text{ -- easy enough}$$

$$k(x) = \ln(x)$$

$$k'(x) = 1/x \text{ -- also easy}$$

$$u(x) = (5x^2 - 2x)$$

$$u'(x) = (10x - 2) \text{ -- simple (the power rule is my friend)}$$

now we plug it all in:

from above so I don't have to keep scrolling up:

$$y = (5x^2 - 2x)e^x$$

$$h(x) = k(u(x)) = \ln(5x^2 - 2x)$$

$$\frac{dy}{dx} = (g'(x)h(x) + k'(u(x))u'(x)g(x)) \times y$$

$$\frac{dy}{dx} = (e^x \times \ln(5x^2 - 2x) + \frac{1}{5x^2 - 2x} \times (10x - 2) \times e^x) \times (5x^2 - 2x)e^x$$

Which is the solution. To move it to look like the answer on the test, we rearrange to put y in front, flip the addition, and combine $1/u(x)$ and $(u'(x) \text{ times } g(x))$:

$$\frac{dy}{dx} = (5x^2 - 2x)e^x \times \left(\frac{e^x(10x - 2)}{5x^2 - 2x} + e^x \times \ln(5x^2 - 2x) \right)$$

finally (phew) we pull out e^x as a common factor and put it in front, getting the same solution as A on the quiz:

$$\frac{dy}{dx} = e^x (5x^2 - 2x) e^x \left(\frac{10x - 2}{5x^2 - 2x} + \ln(5x^2 - 2x) \right)$$